ADVANCED ENGINEERING INFORMATICS

Advanced Engineering Informatics 16 (2002) 99–105

www.elsevier.com/locate/aei

Solving vehicle routing problems by maximum neuron model

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Received 5 April 2001; accepted 30 November 2001

Abstract

In this paper, we propose a new clustering method for the first phase of a two-phase method of the vehicle routing problems (VRPs) and the traveling salesman problems (TSPs). In the first phase, the customers are grouped as several delivery areas for vehicle by using maximum neuron model. In the second phase, the TSPs for each areas are solved by using elastic net model proposed by Andrew et al. Conventional maximum neuron model proposed by Takefuji et al. is not suitable for these continuous problems. But by including a self-organization rule to this model, the solution quality is improved. Our simulation results show that maximum neuron model can achieve to obtain better solutions than other methods for some kinds of problems in VRPs and TSPs. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Vehicle routing problem; Maximum neuron model; Clustering problem; Elastic net

1. Introduction

The single depot vehicle routing problem (VRP) [6] is one of the well-known optimization problems. The problem is to minimize the total length of all routes for each vehicle that has a restricted capacity and starts from the same depot. This problem is widely applied to many real delivery problems. Delivery and pickup operations are the situations that this problem can be applied to. The collection of mail from posts and the operation of school bus services are well-known examples of pickup operations.

The VRP can be stated as follows: a set of L vehicles, with same capacity Q, is located at depot D. Customer i is located at X_i in the two-dimensional map and has a demand q_i. Each vehicle, finding a route which begins at the depot, visits a subset of customers and returns to the depot without violating the capacity constraint. The objective is to minimize the total length of all routes (see Fig. 1).

Due to the complexity of this problem, existing methods are almost exclusively heuristic methods that find an approximately optimal solution. Fisher and Jaikumar [7] classifies the previously proposed heuristics to four types:

(i) tour building heuristics,
(ii) tour improvement heuristics,
(iii) two-phase methods, and
(iv) incomplete optimization methods.

The tour building heuristics are based on the Clarke and Wright method [4]. These methods begin with an infeasible solution in which every customer is assigned to a separate vehicle. In iterative steps, link is combined until capacity constraints are not violated. The choice of a link is motivated by a measure of cost savings. Gaskell [8] and Yellow [19] proposed modified methods of Clarke and Wright method that use modified savings.

In the tour improvement heuristics, the solution begins with a feasible vehicle schedule. By exchanging links, the cost is reduced without violating constraints. These heuristics are based on the Lin [11] and Lin–Kernighan [12] method for the traveling salesman problem (TSP), and Christofides and Eilon [2] and Russell [14] have modified these heuristics for the VRP.

In the two-phase method, customers are first assigned to vehicles before deciding the sequences. In phase two, routes are obtained for each vehicle by solving a TSP. Tyagi [17], Gillett and Miller [9], Fisher and Jaikumar [7] and Christofides et al. [3] proposed two-phase methods. These methods are different for the costs that were used in phase one. The methods in Refs. [9,17] use the distance between customers as cost. In Ref. [17], customers are assigned sequentially to a vehicle using a nearest neighbor rule. Gillett and Miller use a 'sweep' algorithm for phase one. A customer is chosen at random and the ray from the central depot through the customer is swept either clockwise or counter-clockwise. Fisher and Jaikumar [7] formulated nonlinear generalized assignment problem, and introduced approximated delivery

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PII: S1474-0346(01)00006-4

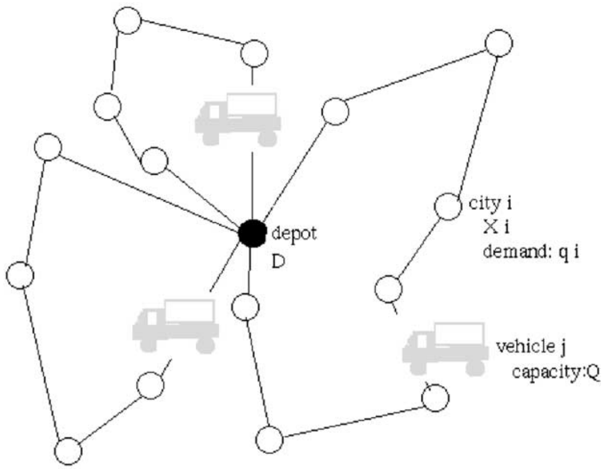

Fig. 1. Vehicle routing problem.

cost. Fisher and Jaikumar method is based on a Lagrangian relaxation in which the multipliers are determined by a multiplier adjustment method described in Ref. [7].

In addition to these four types classified by Fisher [7], a topological approach exists. Elastic net algorithm for solving the VRP proposed in Ref. [18] is based on elastic nets for TSP [5,15]. Durbin and Wilshaw [5] and Simmen [15] indicate that this topological approach can obtain good solution for small size TSPs. However, as this method is a one phase strategy, it is difficult to obtain the best solution for VRPs and large scale TSPs in realistic computational time.

This paper proposes a new clustering method to use as the first phase in the topological approach for the simplification of the problem. This approach is regarded as a two-phase method. In the first phase, the routing costs are approximated for each customer as distances from the *main route* that is mainly traced by each vehicle. This assignment problem is solved by a framework of clustering algorithm that uses maximum neuron model proposed by Takefuji et al. [16]. Takefuji et al. [16] indicates maximum neuron model can solve several kinds of discrete optimization problems efficiently. Amartur et al. [1] shows the model can also apply the segmentation problem of magnetic resonance images. As pointed out in Oka et al. [13], while the model has an advantage of fast convergence, the quality of the solution for the clustering problem is inferior to the Kohonen model proposed by Kohonen [10]. Oka et al. solved this problem by using Kohonen model after using maximum neuron model. In the maximum neuron model phase, an approximate optimum solution can be searched quickly and in the Kohonen model phase, a more precise solution based on the solution of the previous phase can be searched. In this paper, it becomes possible to search a solution continuously by including the self-organization rule to maximum neuron model. Our results show the qualities of the solutions are improved from the conventional maximum neuron model. Apart from the distance minimization clustering problem, this model can also be applied to the cost minimization problem that imposed several constraints like VRP or TSP. Our simulation results also show that our two-phase method for VRPs and TSPs are not inferior to the other conventional models. Comparing Vakhutinsky and Golden [18], Clarke and Wright [4], Christofides and Eilon [2] and Christofides et al. [3], our method could obtain the best solution for two kinds of VRPs in our simulation. Comparing Vakhutinsky and Golden [18], our method could obtain better solution for large scale TSPs.

2. Clustering algorithm

2.1. Strategy of this method

In the first phase, delivery costs should be decided before delivery route is settled. In the clustering algorithm, the costs are approximated as distances from feature lines. The strategy is based on the assumptions that if the vehicle can go and be back along a straight line ideally, it is the most efficient for each driver. In the real distribution of the customers, linear arrangements of the customers can be found for saving costs. Therefore, the clustering problem can be regarded as a self-organization problem to find feature lines from the map.

2.2. Approximation of costs

A cost c_{ij} for assigning vehicle j to the customer i can be defined as the distance from the main route of vehicle j to the customer i.

c_{ij} can be described as follows:

$$c_{ij} = \begin{cases} |N_{ij} - X_i| & \text{if } (M_j - D)(X_j - D) > 0 \\ |M_j - X_i| & \text{otherwise} \end{cases} \quad (1)$$

where M_j is the representative point of the customers visited by vehicle j, and N_{ij} is the foot of the perpendicular from X_i of the line R_j that passes D and M_j, which is called main route. c_{ij} is the distance from the customer to the main route if the customer locates the side of the representative point or the distance from the customer to representative point otherwise (see Fig. 2). V_{ij} means whether vehicle j visits customer i. V_{ij} takes 1 if vehicle j visits customer i otherwise

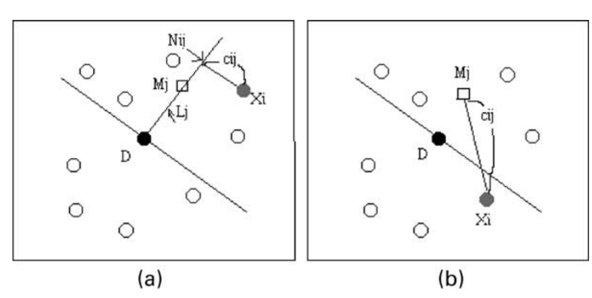

Fig. 2. A cost for visiting city i by vehicle j (a) when X_i locates the side of M_j, and (b) otherwise.

V_{ij} takes 0. By the definition of the VRPs that each customer must be visited only once and that each vehicle have the same capacity constraint, V_{ij} is subject to Eqs. (2) and (3):

$$\sum_j V_{ij} = 1 \tag{2}$$

$$\sum_i V_{ij} q_i < Q \qquad \forall j \tag{3}$$

The total cost can be described as follows:

$$\sum_j \sum_i c_{ij} V_{ij} \tag{4}$$

2.3. Neural network dynamics

An approximate optimal solution of V is selected by using maximum neuron model. To satisfy Eq. (2) constantly, V_{ij} is defined by Eq. (5):

$$V_{ij} = \begin{cases} 1 & \text{if } U_{ij} = \max\{U_{ia}\}\{a = 1, \ldots, L\} \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

U, which is set as random numbers initially, is updated by Eq. (6):

$$U_{ij}(\text{new}) = U_{ij}(\text{old}) + \frac{\mathrm{d}U_{ij}}{\mathrm{d}t} \tag{6}$$

$$\frac{\mathrm{d}U_{ij}}{\mathrm{d}t} = -\alpha c_{ij} - \beta\left(\sum_{a=1}^{\text{num}} V_{aj} q_a - Q\right) \tag{7}$$

α is the coefficient of the cost c_{ij}, β, the coefficient of the constraint of capacity of a vehicle and num denotes the number of the customers. α tends to 0 gradually while the calculation continues (see Eq. (8)):

$$\alpha(\text{new}) = \alpha(\text{old}) - \frac{\alpha(\text{initial})}{T} \tag{8}$$

T is the frequency for updating the parameters. The representative points are changed discretely in the conventional maximum neuron model, but they are updated continuously by using self-organization rule in our model. The representative point of the customers for vehicle j M_j is initialized by Eq. (9) and updated by Eq. (10).

$$M_j(\text{initial}) = \frac{\sum_{a=1}^{\text{num}} X_a V_{aj}}{\sum_{a=1}^{\text{num}} V_{aj}} \tag{9}$$

$$M_j(\text{new}) = M(\text{old}) + \gamma\left\{\frac{\sum_{a=1}^{\text{num}} X_a V_{aj}}{\sum_{a=1}^{\text{num}} V_{aj}} - M_j(\text{old})\right\} \tag{10}$$

where γ is the learning parameter of M_j, and γ towards 0 as the calculation continues using the following calculation:

$$\gamma(\text{new}) = \gamma(\text{old}) - \frac{\gamma(\text{initial})}{T} \tag{11}$$

A continuous movement of the representative point is shown in Fig. 3. The representative point moves toward the real gravity point gradually. V_{ij}, U_{ij} and M_j are updated until V satisfies Eq. (3) and the total cost change equals zero, or the number of iterations reaches the constant steps.

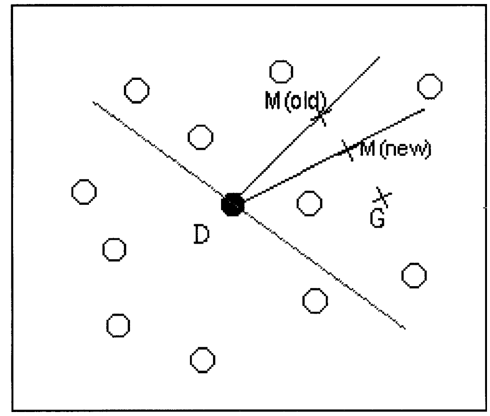

Fig. 3. The movement of a representative point. The representative point, M, moves toward the real gravity point, G, gradually.

2.4. Extension for TSP

The algorithm proposed can be applied to TSPs by modulating following points: (1) D is set to the central point of customers; (2) β, which is the coefficient of the constraint of capacity of a vehicle, is set to 0; (3) after solving each TSPs, the clusters are ordered by solving another TSP whose cities are in accordance with the center points of each cluster (see Fig. 4); and (4) the final solution excludes the depot and connects each route through customers in the order decided in step 3.

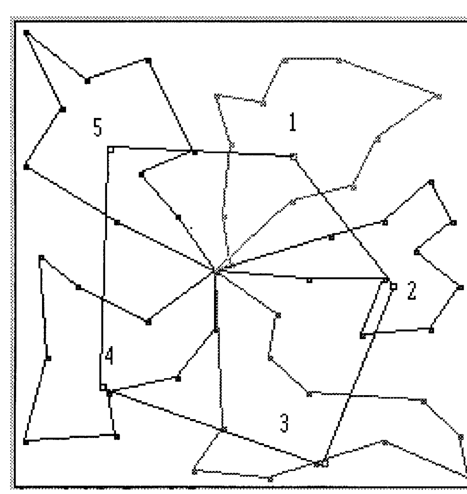

Fig. 4. Decision of the order of the clusters by solving TSPs.

Table 1
Comparison with the conventional maximum neuron model

Problem	Number of cities	Total distance		Number of vehicles	
		Proposed MNM	Conventional MNM	Proposed MNM	Conventional MNM
eil51	50	521	529	5	5
eilA76	75	898	914	10	11
eilA101	100	853	861	8	8
problem4	150	1081	1082	12	12

3. Solving small TSPs

3.1. Elastic nets

In the second phase, small TSPs are solved for the customers assigned to each vehicle by using elastic net [5]. The elastic net is initially set as a small loop, and it is stretched towards the customers with elastic forces like a *rubber band*. Let x_{ij} denote the location of the ith customer assigned to the jth vehicle, and y_{kj} the location of the kth element of jth elastic net. These elements are called beads. The location of each bead is updated by Eq. (12).

$$\frac{\mathrm{d}y_{kj}}{\mathrm{d}t} = \sum_t w_{ik}(x_{ij} - y_{kj}) + K(y_{k+1,j} - 2y_{kj} + y_{k-1,j}) \qquad (12)$$

where w_{ik} denotes the connection weight of x_{ij} and y_{kj}, K denotes the parameter which tends to 0 during the calculation. The connection weight is calculated by using Eq. (13).

$$w_{ik} = \frac{\phi(|x_{ij} - y_{kj}|, K)}{\sum_l \phi(|x_{ij} - y_{lj}|, K)} \qquad (13)$$

where

$$\phi(d,k) = \mathrm{e}^{-d^2/2k^2} \qquad (14)$$

Eqs. (12)–(14) indicate that the nearer bead from the ith customer is preferred to move toward the customer having the gravities of left and right beads. The calculation is continued until the network is converged.

4. Simulation

4.1. Results for VRPs

We approached eil51, eilA76, eilA101 and problem4 in TSPLIB, which are open problems. Sizes of the problems are 50, 75, 100 and 150, respectively. A typical simulation result of each problem is seen in Fig. 5. In Fig. 5(a), each line which connects the depot and each center point shows the main route for each vehicle. In Fig. 5(b)–(d), the line which shows the main routes are left out. Each loop that begins from the depot and returns to the depot shows each delivery route for the assigned vehicle. In the case shown in Fig. 5(a), it converges in 299 steps and it needs five vehicles whose total cost is 522. In the case shown in Fig. 5(b), it converges in 300 steps and it needs 10 vehicles whose total cost is 898. In the case shown in Fig. 5(c), it converges in 300 steps and it needs eight vehicles whose total cost is 862. In the case shown in Fig. 5(d), it converges in 300 steps and it needs 12 vehicles whose total cost is 1099. While each problem has different sizes, the iterative steps for the convergences are almost the same.

We compared the results of proposed model with that of conventional maximum neuron model. Table 1 shows the comparisons of costs and number of the vehicles for the best solutions for eil51, eilA76, eilA101 and problem4. The proposed model improves the solution precision for all kinds of problems. Especially for eilA76, the number of vehicles for our model's solution is fewer than that for conventional model's solution. To minimize the total cost for delivery, it needs to deliver in minimum number of vehicles, but in the conventional model solution, it is not delivered in the minimum number of vehicles. Although it is shown that the proposed model could obtain better solution than conventional model could for most of the results, the result for problem4 is almost the same. As shown in Fig. 5(d), the distribution of each cluster is small. For those small distributions, the self-organization rule is not so efficient. For the large distribution, for example eil51, eilA76 and eilA101, the self-organization rule is efficient to obtain precise solutions.

We also compared the maximum neuron model's solutions with the known best solutions (see Table 2). We compare with elastic net [18] (topological approach) and the savings approach [4] (tour building heuristics), three-optimal method [2] (tour improvement heuristics) and two-phase method [3] (two-phase method). The best solutions of maximum neuron model for eil51 is 521, for eilA76 is 898, for eilA101 is 853 and for problem4 is 1081. The solutions for eil51 and problem4 are the best of all the other methods.

4.2. Results for TSPs

We approached eil101 and a280 of TSP in TSPLIB for our simulation. Each number of the cities is 101 and 280, respectively. The simulation results of these problems are seen in Fig. 6. The best results for our simulation of eil101 is 710 and that of a280 is 3289. Fig. 7 shows the results for

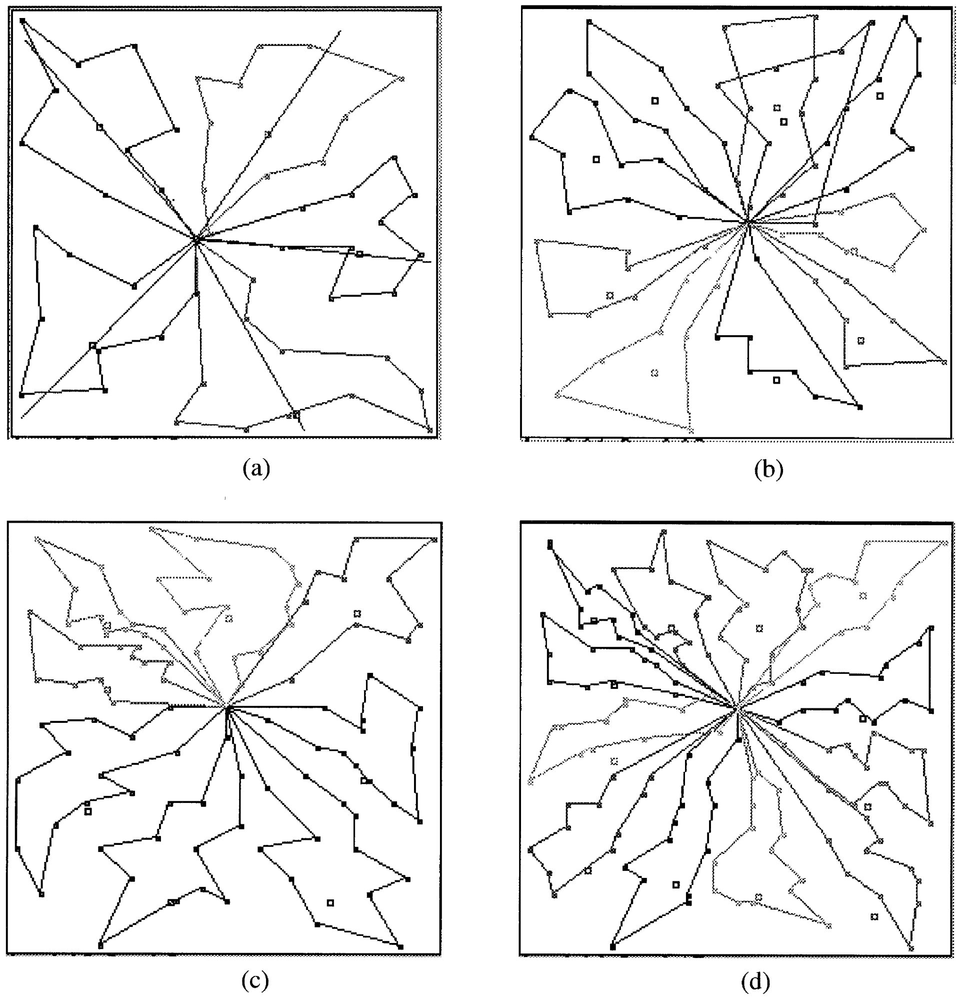

Fig. 5. (a)–(d) Show simulation results of eil51, eilA76, eilA101 and problem4, respectively. (a) $\alpha = 0.02$, $\beta = 0.3$, $\gamma = 0.03$, step = 299, number of vehicles = 5, total distance = 522. (b) $\alpha = 0.01$, $\beta = 0.08$, $\gamma = 0.03$, step = 300, number of vehicles = 10, total distance = 898. (c) $\alpha = 0.02$, $\beta = 0.3$, $\gamma = 0.03$, step = 300, number of vehicles = 8, total distance = 862. (d) $\alpha = 0.04$, $\beta = 0.15$, $\gamma = 0.03$, step = 300, number of vehicles = 12, total distance = 1099.

eil101 and a280 by using one phased elastic net approach. The total distances for eil101 is726 and that for a280 is 3593.

5. Discussion

In our method, the VRPs are solved as a kind of feature extraction problems. In Fig. 5, all of the final clusters have common features that are like long thin ovals. These common features are caused from the self-organization algorithm that searches liner features in the area. In eil51, eil101 and problem4, our model is proper to the distributions of the customers. It is shown in Fig. 5(a), (c) and (d) that the long thin clusters does not cross each other. The comparison of the results shows that our model can obtain

Table 2
Comparison with the other models (method A: proposed maximum neuron model; B: elastic net (Vakhutinsky and Golden) [18]; C: the savings approach (Clarke and Wright) [4]; D: three-optimal method (Christofides and Eilon) [2]; and E: two-phase method (Christofides et al.) [3])

Problem	Number of cities	Total distance					Number of vehicles				
		A	B	C	D	E	A	B	C	D	E
eil51	50	521	560	585	556	550	5	6	6	5	–
eilA76	75	898	–	900	876	883	10	–	10	10	–
eilA101	100	853	–	887	863	851	8	–	8	8	–
problem4	150	1081	–	1204	–	1093	12	–	–	–	–

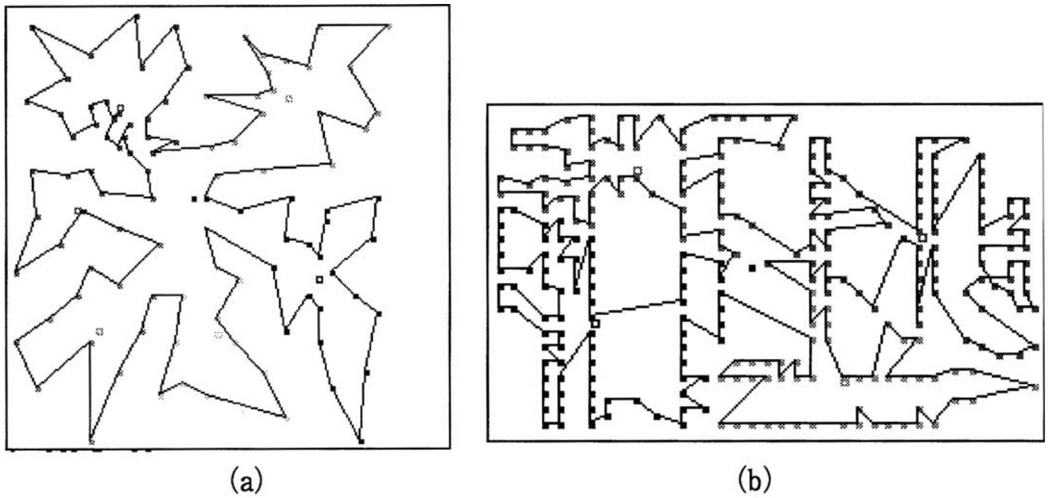

Fig. 6. (a) and (b) Show simulation results of eil101 and a280, respectively. (a) $\alpha = 0.02$, $\beta = 0$, $\gamma = 0.03$, step = 499, number of clusters = 6, total distance = 710. (b) $\alpha = 0.02$, $\beta = 0$, $\gamma = 0.03$, step = 499, number of clusters = 4, total distance = 3289.

the best or the second best solution for these problems. For the solution of eil51, our model takes fewer vehicles than that of elastic net model does, and takes 7% shorter distance than elastic does. Comparing with the elastic net model, by separating VRP into two problems, our method improves the accuracy of obtaining a better solution, and it can adapt to increase of customers. However, in 75 customers' problems, our simulation result does not show the best solution compared to the others. Fig. 5(b) shows that the routes of the vehicles overlap each other, and our method is difficult to obtain the best solution for this pattern. This overlapping solution pattern is because the capacity constraint of each vehicle is difficult to satisfy. In this case, it is difficult to obtain a good solution because the constraint and the cost function are violated. However, this case rarely happens in the real delivery problems because the demands of the customers are usually very small for the vehicle capacity. Therefore capacity constraint has a small weight compared to the delivery cost. By using maximum neuron model, our method obtained best or second best solutions for many problems that are similar to real delivering problems. Also, the experimental results show that the iterative step does not increase if the number of customers increases (see Fig. 5). If this model can be simulated by parallel processors, it can find solutions in $O(1)$ of computational time.

We also show that the proposed method can obtain better solutions for large-scale TSPs than elastic nets can. Although the sufficient condition whether the solution obtained is the best or not cannot be checked without investing all routes, but the necessary condition for the best solution can be checked if the tour has no crossing routes. The simulation results show that we can obtain approximated optimum tours with few crossing routes. That is because the proposed algorithm avoids making a crossing route. However, the problem of how to decide the number of clusters L still remains.

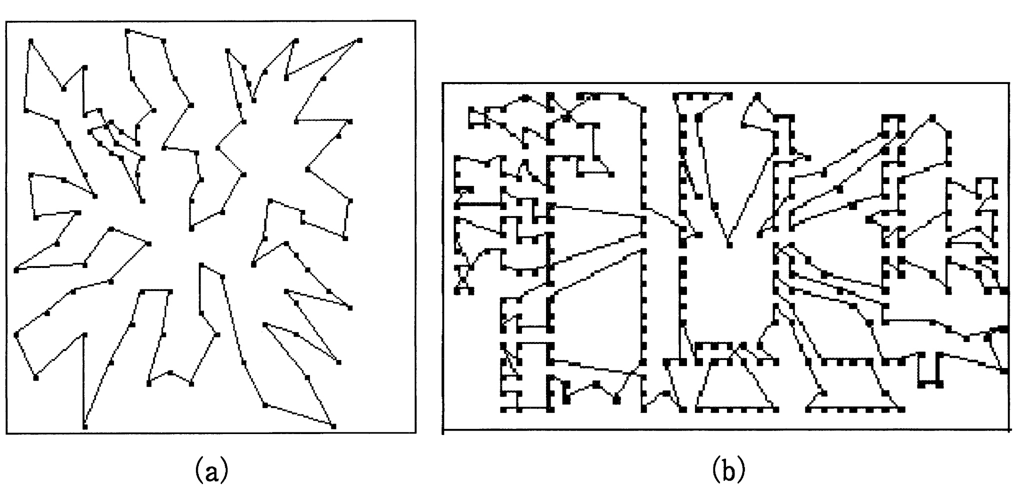

Fig. 7. (a) and (b) Show simulation results of eil101 and a280 by using elastic net, respectively. (a) Total distance = 726. (b) Total distance = 3593.

In this paper, we extend the conventional maximum neuron model to apply topological problems. The comparison of conventional model and our new maximum neuron model shows that quality of the solutions are improved. This extension means that the ability of the maximum neuron model that can solve discrete problems efficiently and the ability of the self-organization method that can solve continuous problems precisely are well harmonized in one model. Cost minimization problems imposed several constraints which could not be solved by using Kohonen model only, but they could be solved by using our model because of the ability of neuron model in solving combinatorial optimization problems. This new clustering method can be applied not only for these delivery problems but also for other topological problems like image segmentation problems that have some constraints.

6. Conclusion

This paper proposed a new clustering method that is based on maximum neuron model. By extending the conventional maximum neuron model, our model can be applied to continuous problems that imposed several constraints like VRPs or TSPs. Our simulation results show that our new clustering model can be used in the first phase of the two-phase approach for VRPs and TSPs and can solve the problems efficiently in terms of the quality of the solutions and the computational time. This extension leads to the area of the problems that maximum neuron model can solve differently.

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